Large-scale structure – witness of evolution

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Abstract

Structures witness the past when they are only metastable or still evolving. Both applies to structures in the universe, in particular fo large-scale, gravitation-dependent structures. They are the subject of the main part of the review. The question is expanded to the structure of theory, too.

1 Structures as Witness – Relics and Fossils

The notion of structure associates with some more or less stably constructed objects in space, a crystal, a building. Structures are determined partly by internal laws, partly by heritage from former structures. The more they represent a stable final state, the less they tell about their history. Structures like crystals are determined by laws that act internally. They are kind of ground states. When they are not degenerate, and a system reaches such a state, it becomes independent of its history. Atoms, thermodynamic states, attractors (chaotic as well as ordinary) belong to this class. They do not witness evolution, they cannot.

But this stability has another side too. In nuclear physics, for instance, time-scales of evolution are rather short. Individual atoms and molecules do not tell about their history.[1] On the other hand, their stability is the reason for their relative concentrations to be frozen, or developing comparatively slowly, and the concentrations on their side tell about the history. This is well known from the stellar production of the different nuclei. The mass distribution of the nuclei is a frozen structure off equilibrium and tells about its history. This kind of telling structures are genuine fossils. They more or less conserve a metastable state created in the past.

But we use not only fossils. Structures in full evolution will depend on their history as well, and tell us about it. As long as the system is still on its path through its configuration space, it characterises to some extent this path, and is witness for its own evolution. Any structure witnesses its evolution as long as it is not in a unique final equilibrium state.

Now for gravitation. Gravitation is peculiar. First, it is weaker than any other force, and time-scales for the evolution are in any case long. Second, the notion of equilibrium is a particularly intricate question for a system of

[1] This property was used by Einstein in order to critizise Weyl's unified field theory that involved history-dependent masses of particles, for instance (Einstein 1921).

gravitationally interacting objects. In a strict sense, they are no thermodynamical systems because of the infinite amount of free energy they formally contain, and their virial state is only a limited substitute (e.g. Kiessling 1989). Thirdly, for unbound systems, the expansion of the universe slows down evolution and even stops it. Short-range interactions are frozen in any case when the dilution of reactants makes the reaction rate smaller than the expansion rate. It is an open question whether gravitational clustering leads to universal distributions, i.e. no more witnessing the past.

For the larger gravitationally interacting systems, the expansion of the universe produces slow-motion the longer the larger the system really is. This makes the recently formed large-scale structure in the universe an ideal witness of the history of this universe, i.e. an ideal witness for cosmology.

Hence, in the following I shall consider the history of the homogeneous universe, that is to be witnessed, the structure, the inhomogeneities that we observe, their evolution, and the questions that can only be followed through numerical simulation. It will end by some remarks extending evolution to the structure of the dynamics itself.

2 The homogeneous universe

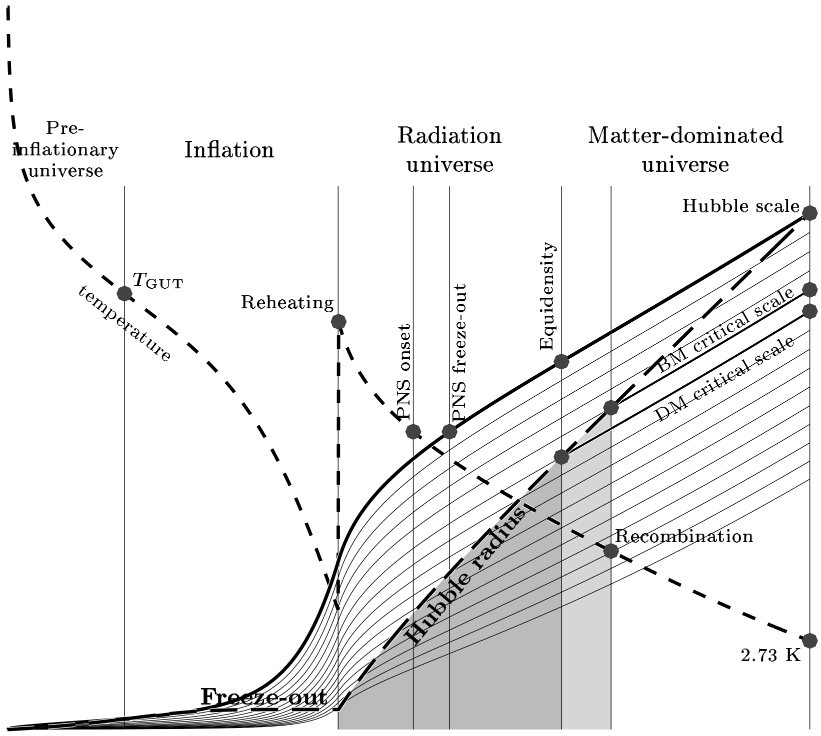
Here we shall shortly consider the history to witness. The evolution of the universe is dominated by its expansion. The universe expands. The frequency shift that increases with distance is interpreted as a kind of Doppler effect. This interpretation has been backed recently by the time dilation of the fading of supernovae (Goldhaber et al. 1996, Riess et al. 1997) in distant galaxies. For instance, tired-light theories do not work any more because the observation of reaction rates does not refer to the inner structure and a possible change of the photons that carry the news. Wave frequencies and reaction rates show the same dilation. This backs the interpretation of the redshift as due to the expansion.

Curiously, the present expansion rate is difficult to determine exactly. Recent determinations use supernovae of typ I in distant galaxies. They give new arguments not only for the expansion rate, but for their change too (Perlmutter et al. 1999). I will not comment the outcome of its analysis, but the question why it is so difficult to know the expansion rate exactly. The reason seems to be that in most of the cosmological phenomena, the expansion rate enters only as a renormalizable scaling or a hardly separable factor. In this respect it resembles the gravitational constant that turns out to be the least precise of all fundamental constants.

The expansion is ruled by matter[2], but not driven. In particular, it is decelerated by matter. We have, however, to be reluctant to relate a velocity to the expansion. There is no velocity, there is only a rate.[3]

[2]Here, matter denotes simply gravitational mass.

[3]The rate is usually given in units of 100 km/s/Mpc, and one writes $H_0 = h \cdot 100$ km/s/Mpc $= h\ 10^{-10}\ \mathrm{a}^{-1}$.



The abszissa is the time in semilogarithmic scale just as the ordinate, that contains lengths and temperatures. The continuous lines indicates the growing of scales by expansion, the normalising scale being drawn a bit bolder. The long-dashed line is the Hubble radius. It is nearly constant during inflation, but grows faster than the scales otherwise. The short-dashed line is the temperature, indicating supercooling as well as reheating after inflation.

Figure 1: Sketch of the evolution of the homogeneous universe

The present expansion rate defines a scale, the present Hubble radius, $R_{\rm Hubble0} = c/H_0 \approx 3000\ h^{-1}$Mpc. By effect of the changing expansion rate, the Hubble radius itself changed in the past, in physical units as well as in scale. In the figure 1, we show the physical size of the scale that equals the Hubble radius today by the continuous line. We should call it the normalizing scale. In comparison with this normalizing scale, the Hubble radius (long-dashed line) increases for matter- and radiation-dominated epochs, stays constant for curvature-dominated epochs, and decreases for vacuum-dominated epochs. For simplicity, we discuss the arguments for matter-domination today, but we redraw everything for other models easily. The expansion is shown by the physical size of the normalizing scale. All other fixed scales expand proportionally. The Hubble scale changes with time and is not proportional to the normalizing scale. As we already stated, the Hubble radius increases faster (with a higher rate) than the fixed scales when the source of gravitation is ordinary matter (i.e. matter with the non-negative pressure that is needed for thermodynamic states).

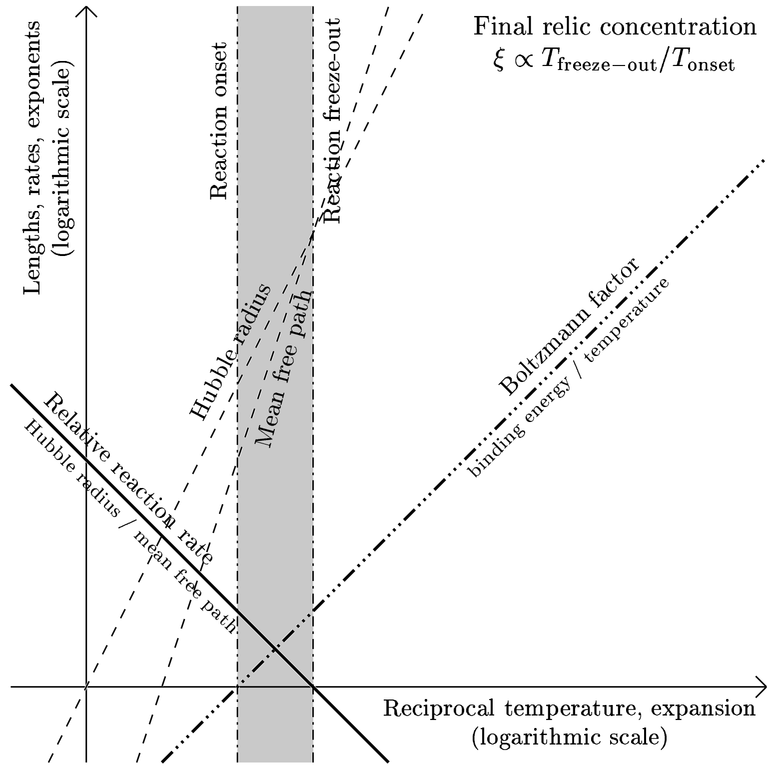
The first conclusion from a prior expansion extrapolated by Friedmann equations was that of a heat bath that has cooled to few degrees Kelvin. Radiation works as a heat bath effectively only if the reaction cross-sections are large enough. Such a state must end by the expansion of the universe, the heat bath will eventually decouple. The decoupling defines the borderline to a transparent universe at later times.

The part of the transparent space that can be observed defines a horizon of a scale of some Gigaparsec (3000 h^{-1}Mpc for a matter-dominated universe). We will return to this topic later. For the moment, we accept the observation of a up to 10^{-3} isotropic black-body radiation as justification for extrapolating the expansion into the far past and for the assumption that the region inside our horizon is homogeneous to an adequate approximation. The earth, the sun, the Milky Way, the local group move through this heat bath, and this motion produces a dipole component of about 10^{-3}. The amplitude of random inhomogeneities is about 10^{-5}. Some time before recombination, the universe must have been dominated by this background radiation, strictly speaking by relativistic particles.

We can extrapolate even deeper in the past till the point where density and temperature must have been these of nucleosynthesis. The processes of primordial nucleosynthesis are characteristic for short-range interactions, that enter a short period out of equilibrium when the temperature falls below the values corresponding to the binding energy, and that ends when the reactants are diluted by the universal expansion so far that the mean free path exceeds the Hubble radius. The resulting concentrations of unprocessed components tell about the conditions at this time, in particular about the evolution of the temperature, expansion and expansion rate. We recall the discussion of the nucleosynthesis bounds on the concentration of baryonic matter, but this is not our topic.

We only show a graph that qualitatively describes the competition between cooling and dilution (fig. 2). Any process of condensation or binding starts when the temperature dropped to that value which corresponds to the binding energy, $\Delta E \approx kT_1 \propto {a_1}^{-1}$. Phase-space factors may correct its value. If the reaction cross-section does not change too much, the reaction rate is determined by the number density of reactants and falls with $(1+z)^3$. This is faster than the fall of the expansion rate $H \propto (1+z)^2$. So at some point $a_2 \propto {T_2}^{-1}$ the reaction freezes out. The width of the allowed interval determines the then frozen concentrations of unprocessed reactants, $\xi \approx T_2/T_1$, like deuterium in the case of the helium synthesis.

Now we have to recall the fact that the standard process just considered leaves only 10^{-19} baryons per photon when the total baryon charge of the universe is zero. This is a 10 orders of magnitude contrast to the observed value of about 10^{-9} baryons per photon. The universe had a non-zero total baryon charge when the nucleosynthesis began. Any explanation of this fact can only be a process that produces a baryon-charged state out of a charge-zero state.



The abscissa is the time or inverse temperature in logarithmic grading. The vertical axis shows times and exponents in the same grading. The Boltzmann exponent grows $\propto T^{-1}$ to start the Process at $kT_{\mathrm{onset}} \approx \Delta E$. The inverse reaction rate grows faster ($\propto T^{-3}$) than the Hubble time ($\propto T^{-2}$). The quotient falls to end the process at $T_{\mathrm{freeze-out}}$

Figure 2: Sketch of the competition between cooling and dilution

That is, we must find processes that destroy symmetry. Grand unification theories of elementary particles provide such a possibility. At some temperature just below 10^{28} K, metastable particles of about grand-unification mass may decay out of equilibrium in baryon-charge asymmetric channels. That means, the expansion must allow extrapolation till this temperature.

However, GUTs must in general accept a monopole problem. When the grand-unification symmetry breaks down, topological quasiparticles (monopoles) are produced in a concentration, that would yield a universe that could never develop into the state we observe. An all-diluting expansion could help at this point, and it is provided by the same theory through the possibility of a large vacuum energy density.

The idea is that a metastable high-temperature vacuum exists that produces an intermediary deSitter expansion (the primordial inflation) when the temperature has fallen below its Curie temperature. It decays finally into radiation and particles of high rest mass. These particles again decay, but with a rate smaller than the cooling rate (which itself is equal to the expansion rate) so that in the non-equilibrium regime asymmetry can arise.

The monopole problem is not generic, but horizon problem and flatness problem are. These two problems and their solution by the inflationary universe have been amply discussed. However, the most appealing and most important feature of inflation is the explanation of the existence of just these inhomogenities that could originate the large-scale structure that we observe today, their being small, and their flat perturbation spectrum, and their distribution being Gaussian. We shall return to this point later. When the first models of inflation were constructed, we were told that the idea would die promptly when a cosmological constant (equivalent to a vacuum density) or a space curvature would be found. But, as E.A.Poe put it, nihil sapientiae odiosius est acumine nimio, nothing is less pleasant to wisdom than too much of acuteness: We have learnt to supplement both into inflationary models (Albrecht 2000).

In short, the universe cools from a high-temperature radiation phase (with a high-temperature vacuum) into a (due to this vacuum) inflationary phase. The unconventionally negative pressure of the vacuum produces an accelerated expansion. The universe is supercooled. Just because of the negative pressure that drives inflation, the high-temperature vacuum is unstable. Its decay into low-temperature vacuum and particles corresponds to a reheating, that must go far enough to produce the particles that can subsequently decay asymmetrically and produce the baryons, but not so far to produce a new monopole problem. The low-temperature ($T < T_{\rm GUT}$) radiation cools down to the temperature where nucleosynthesis can begin. The expansion of the universe stops nucleosynthesis by dilution. The radiation cools till the weight of the massive (or non-relativistic) particles determines expansion and further till uncharged atoms form and the universe becomes transparent. The then quasi-isolated radiation cools till the 2.73 K of today.[4]

We have evidence from the recombination time and from nucleosynthesis, but the deeper past is still in the dark. It is the knowledge about inhomogeneities and their evolution to large-scale structure that enables to tell about the deeper past, in particular about the hypothetical inflation. That is because we can safely calculate the inhomogeneity on a (comoving, expansion-reduced) scale all the time back till it coincides with the Hubble radius in the inflation epoch. The structures we observe today tell about at least some properties of inflation.

[4]It should be noted that the inflationary scenario simplifies the choice of a cosmological model by providing kind of a common initial state of nearly zero temperature. The reverse side of the medal is that the reduction of the set of early histories makes the universe in some sense more improbable: The universe is the more probable, the more virtual prehistories are conceivable. This is the problem of the preinfaltionary universe.

3 The inhomogeneous universe

3.1 Homogeneity of structure

Homogeneity in the universe is apparently homogeneity in the distribution of observable objects, but the deeper reason is the homogeneity in law, together with the assumption that the observed distributions and structures are realisations of homogeneous probability distributions that are on their turn the outflow of generally valid laws. We have to keep in mind that a generally valid law may be hidden by a very large-scale structure that may arise from strong nonlinear interactions of the objects in question with the environment given by the other objects, but gravitation is weak, and we observe some large-scale homogeneity as well. When we think about observable structure, we think first of stars, then of galaxies, then of clusters of galaxies. There is a hierarchy. Stars and galaxies are different by constitution, clusters and groups of galaxies are difficult to discriminate properly, and we find distinctions as long as merging times are short compared to life times. Stars and galaxies are objects that reflect largely a kind of equilibrium between gravitational condensation and other processes, hydrodynamical and radiational. Galaxies are self-determined structures that are defined by the comparability of free-fall and cooling times. Inside most of the big galaxies black holes are supposed to exist. They are responsible for a characteristic velocity profile and they produce virialisation by chaoticity of orbits in their vicinity, in cooperation with or contrast to violent relaxation. Clusters seem to be dominated by gravitation only. There might exist superclusters, although they are difficult to identify, and they seem to be only few (Tucker, Lin & Shectman 1999). In addition, there is lots of diffuse matter between galaxies and clusters, tracing the graviational potential. Thin cold clouds imprint their existence by Lyman-alpha absorption lines in quasar spectra. They seem to condense in the shallower wells of the gravitatiopnal potential and evolve not only by the influence of gravitation, but also by the heating through quasar UV light (Petitjean et al. 1995, Riediger et al. 1998). Thin ionised gas is captured by clusters and shines in X-rays (see Henry 2000). Its evolution can be calculated and compared with observation. The evolution later than $z = 1$ is only marginal. As we shall see, this tells about a low-Ω universe.

3.2 Matter, luminous, dark, and obscure

The gravitational field of structures like galaxies or clusters of galaxies can be measured by satellite motion (rotation velocities of gas clouds), by light deflection (gravitational lenses), by gas temperature (X-ray emission) and velocity dispersion in general.

The most striking feature is the apparent discrepancy with Newton's gravitation: The field of galaxies and clusters is stronger than expected. These facts are usually interpreted like the Uranus orbit that did not exactly follow the calculated path: Matter still unseen. We do not take the observation to test Newton's gravitation, we take it to measure the gravitational mass distributed in the observed objects and call it dark matter. This procedure has its precursor in the calculation and the detection of the planet Neptune through the perturbations of Uranus. It may lead into a trap, as we know from the perihelion motion of Mercury: An unseen planet was considered to be responsible, was christened Vulcan, and observed in 1879 (Oppolzer 1879). Vulcan, however, was spurious, and the perturbation of Mercury indicated the necessary transition to an improved theory for gravitation. Nevertheless, in cosmology only a small community looks for a solution with an improved or changed gravitation theory.

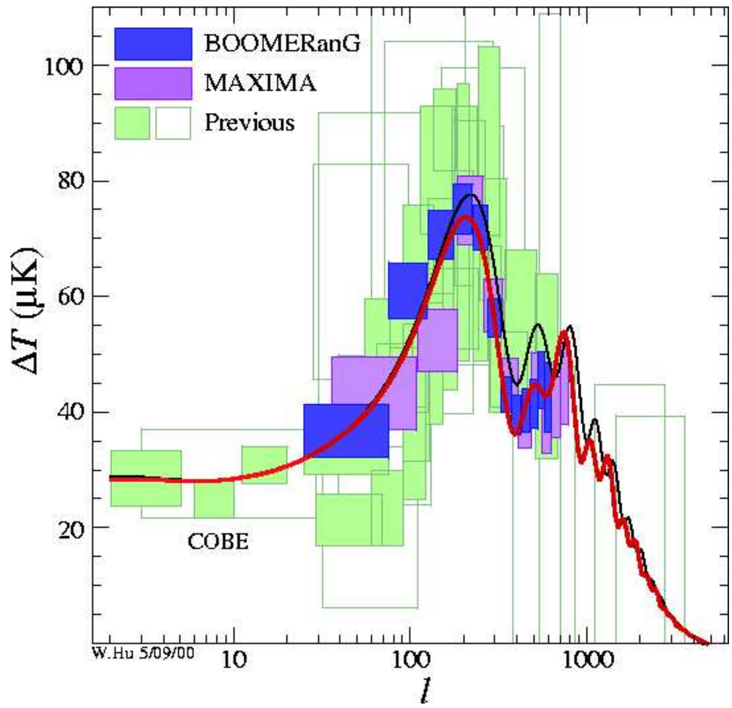
As we shall see, dark matter is not only necessary to keep mass condensed, it is even necessary to restart the evolution of structure on smaller scales early enough. This is a step farther, we do not infer this matter from Newton's gravitation alone, but have to combine with the paradigm of structure evolution under the conditions required by primordial nucleosynthesis. Therefore, this dark matter is not simply dark ordinary matter (MACHOs). It is darker than dark, it is obscure. Nevertheless, it is widely accepted, and there are plenty of theoretical candidates with different appeal and, for their gravitational effect, only slightly different properties: WIMPS, gravitinos, neutralinos. Dark matter might be cold (non-relativistic at matter-radiation equidensity) or hot (relativistic at matter-radiation equidensity) or warm, or a mixture of these. Its density might be critical (as it is used in the so-called standard CDM model that is more and more unlikely to be viable) or subcritical (which requires a cosmological constant and/or space curvature to fit the Friedmann equation). Standard structure formation theories suppose the dark matter to be collisionless, and gravitationally dominant (see Jenkins et al. 1996).

3.3 Large-scale structure

Clusters and galaxies are not distributed at random, but show a still larger structure. Galaxies and clusters are the points that mark this large-scale structure, our topic. These points have a distribution in space. The determination of their positions in the three-dimensional space depends on a good determination of their redshift. By virtue of the Hubble expansion, the redshift is translated into a distance. Without redshift catalogues, no large-scale structure can be studied (see Giovanelli, Haynes 1991, Colless 1999).

The striking feature is the existence of voids. *Giant voids in the universe* was the title of an article in Nature 1978, when first conclusions were drawn that still expected 10^{-4} anisotropies in the microwave background radiation. In spite of the fact that voids are known and are studied, they are bordered by rather a network of filaments than walls. Concentration of matter in the knots is accompagnied by emptying the voids.

Voids slowly get emptier, and become more and more spherical. There must be time enough for the voids to get empty. Besides the void probability new and effective statistics have been found (Müller et al. 2000). The ubiquitous size of voids is $10-40\ h^{-1}$Mpc, but there are voids between superclusters $100\ h^{-1}$Mpc in diameter.



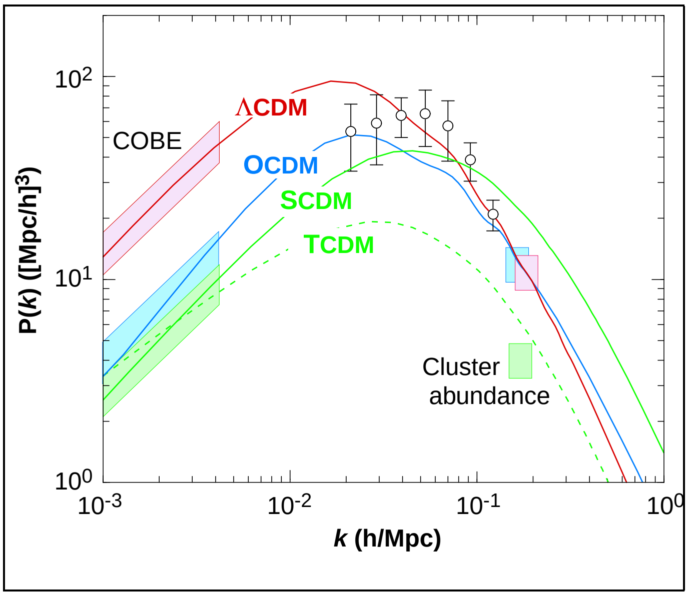
The structures in the spectrum (called acoustic peaks) are the result of the switch-on oszillations of the condensation of baryonic matter. The position of the first acoustic peak indicates consistency with a universes without curvature. The deepness of the first depression and the smallness of the second peak are unexpected. The black curve indicates this expectation from a Standard CDM universe, the grey curve is an adapted ΛCDM model.

Figure 3: Angular spectrum of fireball inhomogeneities, taken from Hu, Fukugita et al. (2000)

Voids are emptied to walls, walls to filaments, filaments to knots. The fastest process should be that from walls to filaments: So fast that filaments are dominant (Bharadwaj et al. 1999).

3.4 The background radiation

Most recent observations of the decoupling epoch by BOOMERanG (Balloon observations of millimeter extragalactic radiation and geophysics) and MAXIMA (Millimeter Anisotropy eXperiment IMaging Array) find the perturbation spectrum in the temperature down to scales of less than 1 degree that correspond to 10 h^{-1}Mpc in the case of an Einstein-deSitter model. The knots in the map of the anisotropies can now be interpreted as the seeds for superclusters. Not of the superclusters which we identify in our cosmic



The abszissa grades comoving wave numbers. COBE measurements of the CMB anisotropy (boxes on left) and measurements of cluster abundance at $z \sim 0$ (boxes on right) impose different quantitative constraints for each model. The COBE-normalized SCDM model significantly overshoots the cluster constraint (lower box at right). The data points with 1σ error bars represent the APM galaxy red shift survey (Peacock 1997). If one assumes bias, then the marks from catalogue evaluation can be shifted to match the model (bias reduction).

Figure 4: Perturbation spectra, taken from Bahcall et al. (1999)

neighbourhood, of course, but far away on the horizon. The spectrum of the inhomogeneities is represented as spectrum of spherical harmonics (fig. 3).

3.5 Measures of homogeneous structure

All measures of large-scale structure suppose it to be the realisation of an underlying probability distribution, with constant mean density and distance-dependent two-point and/or higher correlations. As in kinetic theory, we need some ergodic property in order to identify the averages in our actual point distribution with the averages of the probability distribution.

The simplest way to describe this distribution is the power spectrum. The density field is decomposed into Fourier modes, and the square of their amplitude is the power, $P[k] = \delta_k^2$. The relative dispersion of mass in a logarithmically fixed scale interval on a scale that corresponds to the wave number is given by

$$\Delta^2[k] = k^3 \delta_k^2,$$

so the difference is a mere faktor k^3.

Such power spectra are shown in figure 4. For smaller scales (depending on the size and completeness of the redshift catalogues) it is to be controlled by observation, for very large scales (just below our horizon on the recombination fireball) we have a check with the inhomogeneities in the microwave background. The relation of its temperature anisotropies to density inhomogeneities needs already to be a bit sophisticated for the accuracy of the data. It depends on the cosmological model. So the COBE mark differs between critical-density and other models. If the density is subcritical, the mark shifts to higher values (mainly because of the integral Sachs-Wolfe effect, see Bunn and White 1995) and laterally (because of different sizes of the horizon, Peacock and Dodds 1994). An important point in this kind of spectra is always furnished by clusters of galaxies (see Retzlaff et al. 1998).

4 The evolution of structure in the universe

4.1 The picture

How does the evolution really work? If there were no pressure in matter, and if the Hubble scale would not increase too fast, the condensation would proceed steadily. The density contrast would increase with the expansion parameter until the point where non-linear evolution begins. Pressure has two effects. First, it changes the evolution of the expansion rate, second, it resists condensation, as we know from stellar structure, for instance. It is a good approximation to say that – as far as space curvature and vacuum contributions are not dominant – the amplitudes in the inhomogeneities of the gravitational potential are constant. This statement resembles the theorem that the outer gravitational field of a star is independent of its composition and structure as long as its mass does not vary. The evolution in density will depend on the Hubble radius and on the Jeans radius, which both depend on time.

Calculating the evolution from small inhomogeneities includes averaging of Einstein's equations. In addition, it requires a reason for the existence of such inhomogeneities. For large scales, there is the problem of gauge invariance, to be solved by the construction of gauge-invariant potentials (see Brandenberger et al. 1993). Beyond the Hubble radius, the theory yields a proportional evolution in all components that respects the conservation of inhomogeneities of the potential. Outside the Hubble radius, the short-range pressure effects are not efficient. Amplification proceeds undisturbed. When a scale is overcome by the Hubble radius, it might be different. And now, the distinction between dark matter and baryonic[5] matter becomes important.

[5]Here matter that is coupled to the electromagnetic field.

Dark matter decouples from the heat bath earlier than nucleosynthesis. Cold dark matter is non-relativistic at the end of the radiation-dominated epoch ($kT_{\text{transition}} \approx 10$ eV). As soon as the scale in question is overtaken by the Hubble radius, the radiation pressure begins to resist condensation, at least for small scales that are passed by the Hubble radius early enough. We obtain two critical scales. The first is determined by the Hubble scale at equidensity of radiation and matter. Scales larger than this enter the Hubble radius at times when radiation has lost its gravitational control. Dark matter on smaller scales resumes condensation at this time. The second is determined by the Hubble scale at recombination. Scales larger than this enter the Hubble radius at times when radiation pressure has lost its effect on baryonic matter. Baryonic matter on smaller scales resumes condensation at this time.

There are two regimes. The dark matter component, decoupled from radiation, starts a life of its own on the background of a still radiation-dominated universe. On a scale that falls below the Hubble radius in the radiation dominated epoch the amplification of the radiation perturbations is suppressed and changes to oscillation. With a Hubble scale increasing as $R_{\text{Hubble}}/a \propto 1/(1+z)$, the theory yields for dark matter still condensation, but only a marginal one. The situation changes not before the universe becomes matter-dominated, at matter-radiation equidensity. The Hubble scale then increases as $1/\sqrt{1+z}$ only, and the small-scale dark matter inhomogeneities resume the growth of the large-scale ones that is proportional to $1/(1+z)$ (fig. 5).

Baryonic matter is tightly coupled to radiation till recombination, and although it is non-relativistic, its temperature is determined by interaction with photons. The light velocity determines the Jeans radius, which is of a scale similar to that of the Hubble radius. No further condensation till photons cool and baryons dilute to the point where the electromagnetic interaction rate must fall below the expansion rate. This is the time of recombination of charged particles (that were never combined before). Radiation pressure does not protect baryons from gravitational condensation any more.

4.2 Linear evolution

In first approximation, Fourier components are decoupled. They follow a linear evolution. To begin with, the first-order perturbations to the space-time metric must be carefully reduced for the quasi-gauge freedom of General Relativity, but this is important for very large (larger than the Hubble radius) scales only. Scales below the Hubble radius can be evaluated by Newtonian mechanics.

The equation for the perturbations are of second order, and there exist two differently behaving modes. In the interesting cases, there is an increasing and a decreasing mode. If the two are of the same order of magnitude for some initial time, the increasing mode becomes the only interesting. However, if the parameters of the equation are changing more or less abruptly, both modes will split in two again, and the newly increasing mode may contain contributions from both former modes.

With $\delta \boldsymbol{v} \stackrel{\text{def}}{=} a[t]\boldsymbol{u}$ and $\delta \stackrel{\text{def}}{=} \frac{\delta\varrho}{\varrho}$ we obtain $\dot{\boldsymbol{u}} + 2\frac{\dot{a}}{a}\boldsymbol{u} - \frac{\boldsymbol{g}}{a} + \frac{\text{grad}\delta p}{\varrho_0} = 0$ and $\dot{\delta} + \text{div}\boldsymbol{u} = 0$. This leads to the famous equation

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - \delta(4\pi G\varrho_0 - \frac{c_{\text{sound}}^2 k^2}{a^2}) = 0$$

with the usual Jeans length $\lambda_{\text{Jeans}} = c_{\text{sound}}\sqrt{\frac{\pi}{G\varrho}}$. For the gravitationally dominating component, the evolution proceeds in conformal time, $\frac{\mathrm{d}\eta}{\mathrm{d}t} = \frac{1}{a}$, for scales above the Jeans radius like for scales above the Hubble radius, $\delta \propto \eta^2$. Gravitational potential perturbations are independent of time. All difference lies in the behaviour below the Jeans radius. There are three epochs. In the first, radiation is the dominant component. In the second, dark matter takes over, but baryons are still tightly coupled to the heat bath. In the third, the heat bath puts the baryons free.

Cold dark matter is non-relativistic at the end of the radiation-dominated epoch ($kT_{\text{transition}} \approx 10$ eV). As soon as the Hubble radius passes the scale in question, the radiation pressure begins to resist condensation, but it affects only the heat bath and the baryons coupled to it. The dark matter is not affected, but it is not yet gravitationally dominant. The growth of its perturbation is far smaller than before. Dark matter resumes the former rate when it becomes gravitationally dominant, i.e. when the universe enters the second epoch. Now, the baryons are still coupled to the heat bath, and the Jeans radius is determined by the high pressure of photons ($3p \approx \varrho c^2$). Hence, baryonic and heat bath perturbations are still in an oscillatory regime. In the third epoch, after 'recombination', baryon perturbations not only resume the η^2 growth, but fall in the potential wells prepared by the already grown dark-matter perturbations. These non-linear processes give rise to the rich structure in the background radiation, and contribute to the bias between mass and luminosity of the evolving structures.

The suppression of condensation on smaller than critical scales for some time produces formally a break $\Delta n = 4$ in the power spectrum. This limit, however, is reached for scales so small that they cannot be checked today. The actually observable scales feel only the incomplete reduction in growth near the first critical scale, where the dark-matter perturbations do not really stop condensating, but assume some linear law with logarithmic outcome. This produces the observed slope in the power spectrum.

Damping by diffusion (the diffusion length is the geometric mean of the horizon scale and the mean free path of the photons) and oscillation by pressure modify the expected state of the fireball. Accurate results require a solution of the Boltzmann equation to follow the evolution in detail. But we finally get at transfer functions (Bardeen et al. 1986).

In a dark matter model, baryons now fall into the potential wells prepared by the dark matter, and begin to keep pace with the dark matter condensation. The onset process makes its impression on the microwave background as well as on the actual phenomeology of the condensations, but in the large, condensations appear in the late universe as if they started at matter-radiation equidensity. If dark matter would not be present, the baryonic matter had to prepare its own potential wells for condensation, the time would be too short for simple condensation, and the characteristic point in the power spectrum would indicate a scale larger than the observed one.

As long as the linear approximation on scales that do not interfere is valid, analytical computation may be performed (the differential equations are solved mainly numerically, of course). The nonlinear stage is characterized by interference of the evolution on the different scales, and could be followed analytically only by a full-size theory of gravitation and kinetics. Therefore, one tries the simulation concept. Particles are subject to newtonian gravitation and followed in their behaviour. For large boxes, hundred particles must suffice for a galaxy. This number is raising with the improvements in hardware and software. In particular, successive refinement procedures have been developed.

Primary evolution is that of the dark matter. Presumably, in halos of dark matter, protogalactic blocks form and merge to galaxies. Swimming in dark-matter haloes that provide for the potential wells, galaxies are self-determined structures that are defined by the comparability of free-fall and cooling times. Inside most of the big galaxies black holes are supposed to exist, that are responsible for a characteristic velocity profile and that produce virialisation by chaoticity of orbits in their vicinity, in cooperation with or contrast to violent relaxation. One finds many small and rather compact galaxies that merge between $z = 3$ and $z = 1$. Later than $z = 1$ one finds only marginal evolution in number.

All evolution is affected by environment (morphology-density relation, biased formation). There are only few properties that seem to be independent at large, for instance the Tully-Fisher relation, and fundamental plane relations. At z=1 the bulk of high-surface-density elliptcals have formed and settled down to the fundamental plane (Peacock 2000).

The evolution can yield a transmission of an initial spectrum, but will not explain the initial spectrum. The initial spectrum stems from very early times, and the inflationary universe can make predictions for the spectral distributions. The main feature in the evolution is the suppression of increase for small scales before some critical time.

4.3 The fireball

The angular power of the anisotropies show the signs of the transition phenomena in the recombination era that depend on various physical conditions at that time, and the scales around the Hubble scale at recombination and matter-radiation equidensity time.

The most unexpected property of this spectrum is the lack of prominence in the second peak and the deep minimum between the first two peaks (Jaffe et al. 2000). Very fast, explanations have been constructed. One of them is a pure baryon universe, lacking all dark matter. This explanation, however, may work for this spectrum, but it does not weaken the arguments that force us to accept dark matter.

Perturbations in the temperature of the heat bath are to be assumed as trace of perturbations in the gravitational potential and in the density that were the cristallisation points for the structures that we observe now as galaxies and galaxy-distribution features. The fact that gravitation is a tensor field implies that there are two kinds of perturbations, scalar or density, and tensorial or wave perturbations. The latter do not lead to peculiar structures in the recent matter distribution but may affect observations by effects of the light-deflection type, of course, and they are important in witnessing the very earliest inferable processes in the universe. Tensorial perturbations should be observable through the polarisation of the microwave background. These observations can begin with the future missions MAP (Microwave Anisotropy Probe) and PLANCK. A useful primer is given by Hu & White 1997.

For the moment, we have to content ourselves with the already astonishingly precise observations by BOOMERanG and MAXIMA. The spectra show already resonance phenomena in the scales near and below the Hubble radius on the fireball ($\approx 1\ h^{-1}$Mpc in the case of an Einstein-deSitter model). The recombination is rapid because the first resonance does not exhibit signs of dissipation[6]. The second peak is unexpectedly low with respect to the third. This alone rejects the standard CDM model. Different explanations have been proposed, from delayed recombination (Peebles et al. 2000) to even purely baryonic models (McGaugh 2000) A readable analysis of the decisive parameters has been given by Hu et al. (2000). Claims are that four parameters are important: the position of the first peak l_1, its relative height $(\Delta T_{l_1}/\Delta T_{10})^2$, the relative height of the second peak $(\Delta T_{l_2}/\Delta T_{l_1})^2$, and of the third $(\Delta T_{l_3}/\Delta T_{l_2})^2$. Their separations are set by their harmonic relation to the first peak. They propose a model spectrum with three characteristic l scales, the acoustic l_{A}, the equality l_{eq} and the damping scale l_{D}. Compared with the effort to determine the Hubble constant, this is precision cosmology.

The physical background has been analysed by Hu et al. (1996), the analytically calculable models are found in Hu and Sugiyama (1994). The possible conceptual alternatives are discussed by Kinney (2000).

4.4 Numerical simulations and experiments

The gravitationally dominating role of the nearly free streaming dark matter makes it easy to substitute the matter in the universe by a set of massive points that move under nearly Newton gravitation. The baryonic component that feels pressure by the interaction with radiation is more complicated, and mostly modelled by a fluid in the gravitational field of the dark matter. Without computer simulation, we would be at stake. The first point in computer modelling is to get at mock catalogues that are to be compared with the redshift catalogues at hand. In addition, it is nice to see the structures really develop. Movies can be found at different places.

[6] At $z = 100$, the universe is optically thin even for fully ionized gas.

The initial conditions, i.e. the initial distribution and velocities of the particles are chosen corresponding to the result of the linear approximation. Initial values are continued till some z that depends on the spatial resolution of the calculation. At $z \approx 10$, the first objects form. 256^3 dark-matter particles are standard today, but there are simulations with 1024^3 particles (Colberg et al. 2000). Their total mass depends on the size of the simulation box.[7] The state of art includes the N-body problem with different adaptive procedures to yield higher resolution in denser domains, addition of hydrodynamic approximations for the baryon fluid, inclusion of temperature, secondary radiation, heating of gas. We can simulate Lyman-alpha forests and adapt the hypothetical input to the observed properties of this forest, model broad-line absorbers, objects in small or large halos, merging of objects. In order to find the evolution of smaller objects like galaxies or groups of galaxies, one proceeds a simulation with coarses particles with a local and finer simulation with smaller particles. Their initial conditions are taken from the coarser simulation, then.

A peculiar problem is the accuracy of the quantities and formulas to be used. Newton's force constitutes a problem when it is calculated too well. Then particles for near encounters change their position in phase space very fast when Newton's law is strictly applied. The resolution of force must be kept coarse enough for the step in time. The precision in force has to be topped by the precision in time, in particular in underdense regions. The allowed precision in force is not restricted by the particle density (Splinter et al. 1998, Knebe, Kravtsov et al. 1999).

Another point is that close encounters of the spuriously large particles can lead, through three-body processes, to the formation of unphysical close massive binaries. To prevent this, the force must be softened too.

The use of a numerical simulation requires

- comparison between codes that differ in construction,
- comparison between different realisations of the initial conditions,
- identifications of denser regions in the particle set with observable objects (halos, clusters, galaxies, clouds).

As concerns the first points, many differing codes are in use, e.g. P^3M codes like the HYDRA code (Couchmann et al. 1995) that is used by the VIRGO consortium, and the ART (Adaptive Refinement Tree) code (Klypin 2000). It turns out that they are adequately consistent. The effect of choosing different realisation of initial values should remain in the bounds of cosmic variance. This has been checked too.

One of the actually topical problems is the correct description of merging. Because of the lack of a scale in DM interactions, DM simulations are all similar for the different box sizes. In contrast to this, superclusters look different from galaxies. There is, e.g., a tendency to find in simulations a

[7] The critical mass in a box of size lh^{-1}Mpc is given by 2.7 $10^{11} l^3 h^{-1} M_\odot$.

higher number of satellites to a galaxy than that actually observed for M31 or the Milky Way (e.g. Gottlöber et al. 2000).

The identification problems affect the expectation for the bias in the distribution of the objects in question (The cynic would see that the outcome is more sensitive to bias than to the cosmological model, i.e. that the simulation determines the bias rather than the model). For instance, spirals, ellipticals, and radio galaxies are differently distributed, so they cannot trace mass unbiased. But in low-Ω universes, luminosity should trace matter.

Bias is a conceptionally deep problem. Models of bias developed from a universally given ratio between power in luminosity and power in mass to a ratio depending on scale, on luminosity (i.e. no ratio, but a nonlinear relation), being finally a stochastic variable of unknown distribution. As long as it cannot be modelled by calculation, it is a collection of excuses, and weakens the points that can be made by observation.

5 Non-linear evolution

Non-linear evolution has to be checked by numerical integration of the full set of equations or by numerical simulation of the process. Both methods are sensibly dependent on the correct handling of numerical errors. The latter method is widely used to find appropriate descriptions and has been applied by various authors.

First of all, the non-linear evolution destroys or at least modifies the spectrum found by linear analysis. Non-linear evolution depends on scale and shifts the exponent in the power spectrum for smaller scales, from $\Delta^2 \propto k^{0.8}$ to $k^{1.73}$. Peacock fits a two-power law to the small-scale wing of the spectrum:

$$\Delta^2[k] = \frac{(\frac{k}{k_0})^\alpha + (\frac{k}{k_2})^\gamma}{(1 + (\frac{k}{k_1})^{(\alpha-\beta)/\delta})^\delta}$$

with $\alpha = 0.74$, $\beta = 4$, $\gamma = 3$, $\delta = 0.6$, $k_0 = 0.42h\ \mathrm{Mpc}^{-1}$, $k_1 = 0.057h\ \mathrm{Mpc}^{-1}$, $k_2 = 0.72h\ \mathrm{Mpc}^{-1}$ for $\Omega = 0.3$.

Press and Schechter found from scaling arguments some non-linear clustering functions (Press and Schechter 1974). They are widely used and accepted as standard for comparison, and they are found in many circumstances (Bernardeau 1994). Based on these arguments, the extreme nonlinear evolution is expected to be stable clustering $\bar{\xi}[a, x] = a^3 F[ax]$, critically reviewed by Padmanabhan and Engineer (1997). If stable clustering is valid, $\xi[a, x]$ cannot be independent from initial conditions.

For non-linear analysis of large-scale structure, topological measures become more and more important: Minkowski functionals (Mecke et al. 1994, Schmalzig et al. 1999), void distribution functions (Müller et al. 2000), filament and shape statistics (Dave et al. 1997), or Voronoi statistics (Liebscher 1998).

The topological studies show to some extent that the anisotropic gravitational condensation is consistent with the non-linear and non-Gaussian properties of the late condensation. This sounds beautiful, but I believe that the topological measures are far too sensitive for such a statement. I only recall the fact that just these topological analyses can be used for image recognition.

6 Inflation

After having seen so much about the present work in following the recent evolution, we come back to our topic, the far past to be witnessed. Basically all presentations of the subject speak about inflation. The point is that independent of the particular model that can be adopted for the primordial states of the universe, the inhomogeneities tell about them, in particular about the begin of the last radiation-dominated evolution.

The important fact is the existence of a kind of conservation theorem for the perturbations of the potential

$$\zeta = \Phi + \frac{2}{3}\frac{\Phi + \frac{\mathrm{d}\Phi}{\mathrm{d}a}}{1 + \frac{p}{\varrho c^2}} \qquad \frac{\mathrm{d}\zeta}{\mathrm{d}t} = -H\zeta(o[\frac{v^2_{\mathrm{acoustic}}}{L^2 H^2}] + o[\frac{kc^2}{a^2 H^2}])$$

Before a scale is passed by the Hubble radius, its perturbation needs to be followed by linear approximation of the general-relativistic equations. Hence, we can infer the state of the perturbations at all times of the last period where the given scale exceeds the Hubble radius from the observations today, if we can reduce the observations to the time when the Hubble radius passed the scale. In the inflationary scenario, all scales that we observe today leave the Hubble radius during inflation. The observations today yield information about this time, the earliest events that are accessible by simple theory.

The question of wherefrom the inhomogeneities at the beginning of the last radiadion era were produced finds a preliminary answer in the inflation scenario. However, inflation does not *produce* structure but *reduces* structure to the quantum minimum. Without inflation, inhomogenities should be large, too large for the observed small inhomogeneities on the background. Inflation dilutes inhomogeneities, the universe becomes supercooled, and the remaining perturbations are the zero-point motions on all degrees of freedom. Their spectrum is expected to survive. The spectrum remains unchanged outside the Hubble radius. That is, the determination of the perturbation spectra is a measurement of the physical conditions during inflation, i.e. the deepest observation we can think of if inflation is the correct concept, of course (Copeland et al. 1993, Liddle 1994, Lidsey et al. 1998).

The end of inflation is a phase transition. It should not be strange to expect also some kind of non-linear structure formation. Pattern formation has been considered for the far off equilibrium reheating at the end of inflation, but the results are still controversial (Sonnborner and Parry 2000). The pattern formation is expected to lead to a distinctive, but probably low-amplitude peak in the gravitational wave spectrum, that could be connected with a controversial excess power at 100 h^{-1}Mpc.

7 Expansion on generalisations

Structure is nothing exceptional, but the rule. No observation can be made without structures that can be identified. Dually, there is no observation except that of rule, i.e. homogeneity in a generalized understanding. We are led to a concept of homogeneous structure, eventually expanded to self-similar structure. The history of structure determination is a history of moving out the homogeneity scale. Cusanus' concept and the cosmological principle are the extreme extrapolations of this experience.

The most obvious structure is structure in space, but we should be aware that this is a very limited notion. First, a structure is always spatio-temporal, i.e. a history of an evolution. The stability in time is only a peculiar property. In contrast to astronomy, physics can manipulate evolution in the laboratory. In physics, structure is found in the laws that rule the possible runs of evolution. This generalized structure contains for instance the concept of action extrema, of evaluation of evolutionary paths. It contains also the kind of constructing action integrals by metrics of space-time. The evolution of ordinary structure is understood as exemplification of a metastructure in the manifold of hypothetical virtually possible structures, that is a theoretical law.

This is only to remember that the existence of a space-time structure itself may not be universal. The domain with an identifiable time, or light cone, may have a fixed boundary or a region of approximate validity. Then, the existence of a light cone might indicate an even deeper structure, that could be described by quantum gravity (Horowitz 2000, Kiefer 1999, Rubakov 1999), including string theory (Easson 2000, Gasperini 2000, Sen 2000, Requardt 2000), quasiclassical reduction schemes (Bleyer and Liebscher 1995), and a new view on quantum gravity (Barbour 1999, Barbour and O'Murchadha 1999, Barbour, Foster, O'Murchadha 2000).

Acknowledgements

This paper is the resumee of a lecture at the Annual Meeting of the *Astronomische Gesellschaft.* I thank for all discussions there, but in particular for the support in the process of preparation and for the many hints I received from S.Gottlöber, J.P.Mücket, and V.Müller.

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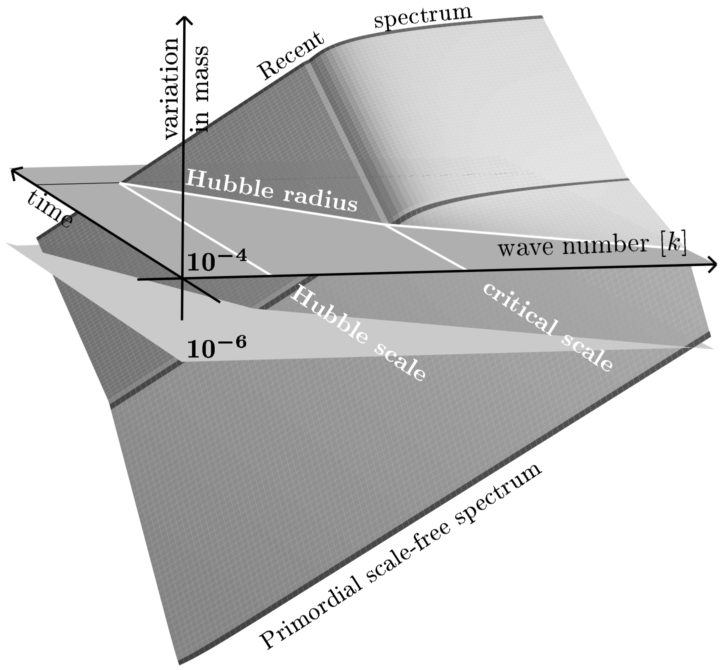

The surface shows the linear evolution in time over the observable scales. The abszissa grades comoving wave numbers, into the depth goes time, the vertical axis is the power times k^3. Before crossing the Hubble radius and before matter-radiation equidensity the spectrum is nearly featureless ($\Delta^2 \propto k^4$) and increases as $(1+z)^{-4}$. We see the region of suppression and the free increase in the matter-dominated epoch as $(1+z)^{-2}$. On crossing the Hubble radius, the value of Δ^2 is nearly the same for all scales.

Figure 5: Sketch of the linear evolution