Wentzel's Path Integrals

by SALVATORE ANTOCI[1] and DIERCK-E.LIEBSCHER[2]

Abstract

Quantum mechanics can be arrived at in three ways, as Heisenberg, Schrödinger and Feynman did respectively. For the last way, an unknown (i.e. forgotten) forerunner exists, that we have found in two papers by Gregor Wentzel, published before the famous works by Heisenberg and Schrödinger, and contemporary with the fundamental works of L. de Broglie. In that paper, one can find the basic interpretation of the action integral as a phase together with the concept of interference of all virtual paths – not only the classical solutions – and the interpretation of the result of the interference as the amplitude of a transition probability, both given by Feynman twenty years later, following Dirac.

There is clear-cut evidence that Feynman's path integral method, with its ground-breaking inclusion of the virtual paths in the formula for the amplitude of the transition probability, has an early precursor. In two fundamental, but presently forgotten papers of 1924 (Wentzel, 1924a, 1924b) Gregor Wentzel anticipated the decisive features of Feynman's method not in vague terms, but by providing a precise mathematical structure that closely matches Feynman's later findings.

One of the reasons why the value of Wentzel's idea was not recognized in 1924, and why the evident link between his proposal and Feynman's approach remained undetected up to now, was the very title of his first paper. Wentzel attempted to overcome the contradiction, existing in theoretical optics, between the wave theory of interference on one side, and the quantum

[1]Dipartimento di Fisica "Alessandro Volta" dell' Università degli Studi di Pavia, Via Bassi 6, I-27100 Pavia, phone +39-382-507-486, fax +39-382-507-563

[2]Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482 Potsdam, email: deliebscher@aip.de, phone +49-331-7499-231, fax +49-331-7499-309

theory of spectral lines on the other side, by interpreting the interferences as the offspring of underlying quantum-statistical laws. As a consequence, his two mentioned papers of 1924 were always read as pertaining to the realm of optics, and the general relevance of the concepts introduced by him – based exclusively on the most consolidated methods of Hamiltonian mechanics – remained hidden.

According to Wentzel (Wentzel, 1924a) the most important foundation of the quantum theory was the law that an atomic system cannot radiate if it finds itself in what he calls a mechanical state, i.e. a state in which the laws of classical mechanics are obeyed. But not only the acts of emission and absorption are "non-mechanical", since the very presence of light propagating through a transparent medium will cause non-mechanical perturbations in the atoms involved in the process.

Therefore, an invariant measure of the deviations of the intra-atomic motions from the ones prescribed by Hamiltonian mechanics must be produced. To this effect, Wentzel considers the canonical co-ordinates β_k and the conjugated momenta α_k associated with the atomic systems involved in the propagation of light. For the sake of simplicity, he chooses canonical coordinates whose conjugate momenta are constant in the mechanical states, and defines the sought-for invariant measure of the perturbation caused by the light quantum through the integral

$$\int \sum_k \beta_k \mathrm{d}\alpha_k \ .$$

This integral is extended over the particular path that corresponds to the deviation from mechanical behaviour which occurs when the light quantum goes from an emitting atom E to an absorbing atom A in some way, not constrained by any equation of motion. In keeping with ideas that "were in the air" at that time (deBroglie, 1923), but exclusively for real, i.e. mechanical paths, Wentzel imagines that to such a virtual propagation of the light quantum one must associate a phase

$$\phi = \frac{1}{h} \int \sum_k \beta_k \mathrm{d}\alpha_k \ . \tag{1}$$

This phase provides the sought-after bridge between quantum behaviour and wave-like phenomena. In fact, since the light quantum can go from the emitting atom E to the absorbing atom A along any one of the previously defined paths, one can build the complex amplitude $\mathbf{F}$ as

$$\mathbf{F} = \sum_s \mathbf{f}_s \exp[2\pi \mathrm{i} \varphi_s] \ ,$$

where φ_s is the phase associated to the s-th path. Since Wentzel wants to adhere in some way to the correspondence principle, he multiplies each exponential by the vector amplitude $\mathbf{f}_s$ of the classical wave that he thinks associated with the s-th path. In reading today his papers, we can abstract from this ingredient without spoiling the essential theoretical structure that he has envisaged. We should also remind that the methods of functional analysis had been invented just one year earlier by Wiener (Wiener, 1923) and surely were not common wisdom for the theoretical physicists.

Wentzel's argument then proceeds to interference. If light would behave in an entirely classical way, the overall probability for a light quantum to go from E to A should be

$$\mid \mathbf{F}_0 \mid^2 = \mid \sum_s \mathbf{f}_s \mid^2 \ \ ,$$

a quantity which Wentzel calls the a-priori probability. Since however light has a quantum behaviour, expressed by the quantum phase, eq.(1), the overall probability for the previously considered process is not equal to the a-priori probability, but it is J times that value, where

$$J = \frac{(\sum \mathbf{f}_s \mathrm{e}^{2\pi \mathrm{i} \phi_s})(\sum \mathbf{f}_s \mathrm{e}^{-2\pi \mathrm{i} \phi_s})}{\mid \sum \mathbf{f}_s \mid^2} \ . \tag{2}$$

It is remarkable that Wentzel finds himself quite at ease with the intrinsically complex nature of his transition amplitude, while for instance Schrödinger (Svchrödinger, 1926b) resisted for some time the intrinsically complex nature of the wave function discovered by him. As concerns the probabilistic interpretation of eq.(2), it was quite obvious to Wentzel. At the very beginning (Wentzel, 1924a) he clearly states his intention to produce a general law for the probabilities of emission and absorption processes of light quanta, which where used since the time of Einstein's derivation of Planck's radiation law, but without making closer assertions about them.

The phase introduced by Wentzel has a very close kinship to the one later considered by Feynman, i.e.

$$\varphi_{\mathrm{F}} = \frac{1}{h} \int \mathcal{L}[q, \dot{q}, t] \mathrm{d}t \ .$$

If one interprets Wentzel's proposal of the first paper as defined in the so-called extended phase space, the kinship becomes an identity (Antoci & Liebscher, 1996) apart from a term common to all paths, which is irrelevant. In the appendix of his second paper Wentzel changes the definition of the phase, and his expression, given in action-angle variables, coincides already in ordinary phase space with the one given by Feynman. Therefore the claim made at the beginning, that Feynman's path integral method was anticipated in its decisive features in the two papers published by Wentzel in 1924 seems fully justified.

Historical note

K.F.Herzfeld immediately understood the importance of Wentzel's approach for the problem of dispersion (Herzfeld, 1924). Wentzel himself tried to solve the dispersion problem in a third paper (Wentzel, 1924c). However, he did not really calculate a path integral, but only used the general idea of his interpretation of the correspondence principle to get a new formula for the susceptibility, which up to some factor, and in a certain approximation, turned out to be the same as the one Kramers and Heisenberg found in their famous paper (Kramers & Heisenberg, 1925). However, Wentzel's formula was more complicated, presumably in an unnecessary way. This was explicitely stated by Kramers and Heisenberg. Afterwards, Wentzel's approach was cited by Landé (Landé, 1925) and rejected by a causality argument, that would destroy even Feynman's approach if it were correct. The last citation we know of is the Handbuch article of Wolf and Herzfeld (Wolf & Herzfeld, 1925), where Wentzel's approach is reviewed and his dispersion formula is cited as one of the approaches alternative to that of Kramers and Heisenberg.

Afterwards, Wentzel's papers of 1924 were forgotten, apparently also by Wentzel himself, who quickly adhered to Schrödinger's equation (Wentzel,

1926) and to Born's probabilistic interpretation (Born, 1926a, 1926b, Wentzel, 1927) of the latter. Wentzel never came back to his papers of 1924.

Dirac's works which inspired Feynman (Dirac, 1933, 1935, 1945) do not contain any hint to a quantum phase formulation, although they state the possibility of a Lagrangian formulation of quantum mechanics, implicitly containing path integrals.

In 1964, T.S.Kuhn (Kuhn, 1964) made several interviews while working for the foundation of the Archive for the History of Quantum Physics (AHQP). He had retrieved a letter from R.H.Fowler to N.Bohr dated April 29, 1925 in which Fowler had enclosed a five-page note by Dirac dealing with an interpretation of "Wentzel's phase". Because the note was lacking, Kuhn asked Dirac about that issue, that seemingly had interested him during the years 1924 and 1925. Dirac advised him to ask Wentzel himself about that phase. In the interview with G.Wentzel, Kuhn reminded of the open question. Wentzel answered that he intended to do something fundamental, but that nobody cared about it except very few physicists, in particular M.v.Laue. Kuhn stressed to Wentzel the isolated position of his three papers of 1924 with respect to his later work, but Wentzel had nothing to add.

Works about the history of quantum mechanics neglect or misunderstand Wentzel's papers of 1924. Even F.Hund (Hund, 1984) does not seem aware of the relevance of Wentzel's papers, although he liked to put the question whether quantum mechanics could have evolved differently. The volumes of Mehra and Rechenberg (Mehra & Rechenberg, 1982) contain only a peripheral citation in connection with the note written by Dirac, but up to now this note has not been found[3]. The note and the letter by M.v.Laue mentioned by Wentzel in the interview seem to be of high scientific importance.

Acknowledgements

We acknowledge the kind help of the Niels Bohr Archive, in particular the attentive cooperation of Felicity Pors. We also thank H.Rechenberg and J.Mehra for helpful remarks.

[3] Communication by F.Pors, Niels Bohr Archive.

References

Antoci,S., Liebscher,D.-E. (1996). The third way to quantum mechanics is the forgotten first. *Annales de la Fondation Louis de Broglie* **21**, no.3.

Born,M. (1926a). Zur Quantenmechanik der Stoßvorgänge. *Zs.f. Physik* **37**, 863-867.

Born,M. (1926b). Quantenmechanik der Stoßvorgänge. *Zs.f. Physik* **38**, 803-827.

DeBroglie,L. (1923). Ondes et quanta. *Comptes Rendus Acad.Sci.* **177**, 507-510.

Dirac,P.A.M. (1933). The Lagrangian in Quantum Mechanics. *Phys.Zs. Sowjetunion* **3**, 64-72.

Dirac,P.A.M. (1935). *The Principles of Quantum Mechanics*, 2.ed, Clarendon Press, Oxford.

Dirac,P.A.M. (1945). On the analogy between classical and quantum mechanics. *Rev.Mod.Phys.* **17**, 195-199.

Feynman,R.P. (1948). Space-time approach to non-relativistic quantum mechanics. *Rev.Mod.Phys.* **20**, 367-387.

Herzfeld,K.H. (1924). Versuch einer quantenhaften Deutung der Dispersion. *Zs.f. Physik* **23**, 341-360.

Hund,F. (1984). *Geschichte der Quantentheorie*, Birkhäuser, Zürich.

Kramers,H.A.,Heisenberg,W. (1925). Über die Streuung von Strahlung durch Atome. *Zs.f. Physik* **31**, 681-708.

Kuhn,T.S. (1964). *Interviews*, Archives for the History of Quantum Physics.

Landé,A. (1925). Zur Quantentheorie der Strahlung. *Zs.f. Physik* **35**, 317-322.

Mehra,J., Rechenberg,H. (1982). *The historical development of quantum theory*, New York, vol.4, p.108.

Schrödinger,E. (1926b). Quantisierung als Eigenwertproblem. *Ann.d.Physik (Lpz.)* **81**, 109-139.

Wentzel,G. (1924a). Zur Quantenoptik. *Zs.f. Physik* **22**, 193-199.

Wentzel,G. (1924b). Zur Quantentheorie des Röntgenbremsspektrums. *Zs.f. Physik* **27**, 257-284.

Wentzel,G. (1924c). Dispersion und Korrespondenzprinzip. *Zs.f. Physik* **29**, 306-310.

Wentzel,G. (1926). Eine Verallgemeinerung der Quantenbedingung für die Zwecke der Wellenmechanik. *Zs.f. Physik* **38**, 518-529.

Wentzel,G. (1927). Zwei Bemerkungen über die Zerstreuung korpuskularer Strahlen als Beugungserscheinung. *Zs.f. Physik* **40**, 590-593.

Wiener,N. (1923). Differential-Space. *J.of Math.& Phys.* **2**, 131-174.

Wolf,K.L., Herzfeld,K.F. (1928). Absorption und Dispersion, Ziffer 52 *in Geiger,H., Scheel,K. (eds.): Handbuch der Physik* **20**, Springer, Berlin.